Pierre
Mendes
Consultants

Structural Design Report

Project	University Ave. Pedestrian Overpass
Location	Waterloo ON, Canada
Engineers	Mapendo Pierre Ngilinga de Carvalho Nicolas Mendes Gomes
Prepared for	Andrea Atkins Studio
Date	18 December 2020

Table of Contents

Pierre
Mendes
Consultants

1 Introduction

Pierre Mendes Consultants Ltd. has been commissioned by AA Studios to provide the associated structural engineering design and documentation for a new pedestrian overpass on the University of Waterloo campus. This report has been prepared to showcase the proposed structural engineering analysis.

1.1 Site

The bridge passes over University Avenue, connecting Carl A. Pollock Hall with the staircase leading to parking lot A.

Figure 1.1 Google Maps street view of the existing bridge

1.2 Design Constraints

Key constraints imposed by the clients are summarized below.

- 2.2 m wide pedestrian walkway
- Must support a load of 10.2 kPa
- Piers must exist at the same locations as existing ones
- No individual structural members can exceed 15 m in length
- The bridge must employ a funicular arch shape and/or cable system

1.3 Reference Documentation

The concept design presented in this report is based on the following documentation:

- Structural Elevation drawings prepared by Giffels Associates Limited

Full drawing set found in client's project document (Atkins, 2020).

2 Proposed Structure

2.1 Schematics

The proposed bridge consists of a series of 4 pairs of crossed arches, spanning 5 "Y" shaped piers at the same locations as the current supports, with the exception of the outermost piers, placed near the ends of the overpass.

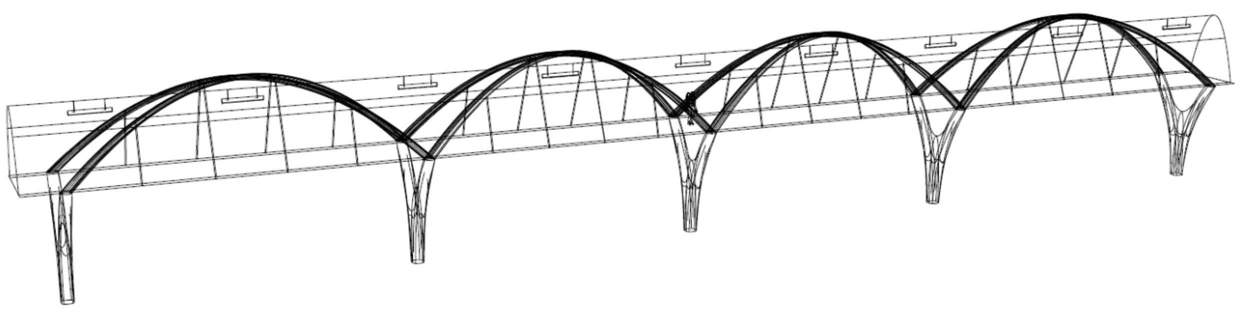


Figure 2.1 Wireframe of proposed pedestrian bridge design

2.2 Motivation

The motivation behind this concept is twofold: practicality and aesthetics.

Practicality

By keeping the piers at the same location as the previous bridge, we are able to keep horizontal spanning distances to a minimum. This reduces the scale of our bridge and increases feasibility. Without any additional support, the horizontal platforms will tend to deflect downwards at their midspan. This means additional support provided between the piers could reduce said deflection and redirect the load in the piers.

Aesthetics

For the choice of intermediary structural support to counter midspan deflection, we decided to go with arches for their clean and modern feel. By slitting the top of the piers into "Y"s and connecting them to the base of the arches, a sense of continuity and fluidity is created. The arches are crossed for a more innovative design, and a pedestrian walkway with a curved roof slides underneath the arches and vaguely mirrors the shape and feel of the new waterloo ION light rail further down the street.

3 Structural Analysis

The following section outlines the scope of the analysis performed on the proposed bridge, the key assumptions made in the process, and a summary of the results. For a more comprehensive walkthrough of the calculation, see Appendix A: Calculation Package.

3.1 Scope

This report focuses on four components of the bridge: the arches, cables, beams and piers.

The goal of this structural analysis is to compute the material properties each component requires to ensure the safety and longevity of the pedestrian overpass. Based on these properties, appropriate sizing is determined, and suitable products are selected from credible sources where applicable, all with a factor of safety of 1.5.

To this end, the following analysis techniques are used.

- General Cable Theorem
- Maximum Tension in Cable System
- Cable Sizing
- Euler's Column Formula
- Buckling and Yielding Failure
- Von Mises Theory of Failure
- Principal of Virtual Work for Deflection and Rotation of Beams
- Maximum Compressive Strength of Arches
- Principal and Maximum In-plane/Out-of-plane Shear Stresses
- Planar State of Stress

3.2 Assumptions

Listed below is a summary of the key assumptions for each bridge component. Details on the justification or impact of such implications can be found in the step-by-step break down of the analysis in Appendix A: Calculation Package.

Category	Assumptions
Arches	Negligible self-weight
	Pinned at support and midspan (three pinned)
	Lateral bracing between cable connections take all lateral force generated by cable tension
Cables	Negligible self-weight

Beams	Negligible self-weight Supported pedestrian traffic and snow loads Pinned supported at ends Allowable deflection of span / 300
Piers	Negligible self-weight Pin-connected to the bridge All tension is transferred to embedded steel rods

3.3 Results

Tabulated below is a summary of the results for each bridge component. For full calculations, see Appendix A: Calculation Package.

Arches

Material	structural steel
Cross-section	round HSS
Outer radius	$r_{out} = 88.9\ mm$
Thickness	$s = 6.35\ mm$
Funicular arch heights	$h_1 = 2.93\ m$ $h_2 = 4.40\ m$
Parabolic approximation	$y = -0.07(x - 7.96)^2 + 4.4$
Max thrust**	$148.87\ kN$
Max bending moment*	$655.0\ kN \cdot m$
Max shear force**	$137.30\ kN$
Max moment of inertia*	$I = 1.41 \times 10^6\ mm^4$

Cables

Material	high strength steel
Cross-section	round
Diameter	$d = 20.1\ mm$
Max tension	$T_{max} = 73.96\ kN$

Beams

Material	structural steel
Cross-section	wide flange
Width	$w = 102\ mm$
Depth	$h = 206\ mm$
Thickness	$s = 6.2\ mm$
Max deflection*	$\delta_c = 6.69\ mm$
Max rotation**	$\theta = 0.38°$
Planar state stresses†	$\sigma_B = 131.06\ MPa$ $\tau_{xy} = \tau_{xz} = \tau_{zy} = 0$ $\sigma_y = \sigma_z = 0$ $\sigma_x = 131.06\ MPa$
Principal Stresses*	$\sigma_1 = 131.06\ MPa$ $\sigma_2 = 0$
Max in-plane stress	$\tau_{in} = 65.53\ MPa$
Max out-of-plane stress	$\tau_{out} = -65.53\ MPa$

Piers

Material	concrete
Cross-section	square
Widths	$w_{arms} = 132.7\ mm$ $w_{base} = 143.5\ mm$
Max axial loads	$P_{arms} = 638.6\ kN$ $P_{base} = 1030.0\ kN$

---

* calculated at mid-span
** calculated at supports
† measured 100mm below neutral axis

4 Renders

Figure 4.1 Interior render

Figure 4.2 Exterior Render

5 Conclusion

In conclusion, the results generated in this report demonstrate that the proposed bridge is able to support all required loading scenarios with a factor of safety of 1.5.

Bibliography

Atkins, A. (2020, December 18). AE205 Final Project. University of Waterloo.

Engineering Toolbox. (2008). *American Wide Flange Beams ASTM A6 in metric units*. Retrieved from Engineering Toolbox: https://www.engineeringtoolbox.com/american-wide-flange-steel-beams-d_1318.html

Leet, K., Uang, C.-M., Lanning, J., & Gilbert, A. (2018). *Fundamentals of Structural Analysis*. New York: McGraw Hill Education.

Steel Tube Institute. (2020). *Dimensions and Section Properties ASTM A1085*. Steel Tube Institute.

Appendix A: Calculation Package

Legend key finding

Arches

1) Tributary Area

The overall geometry and member sizing of the arches will depend on their loading. To determine that we first determine the loads each cable is holding using their **tributary area** illustrated below.

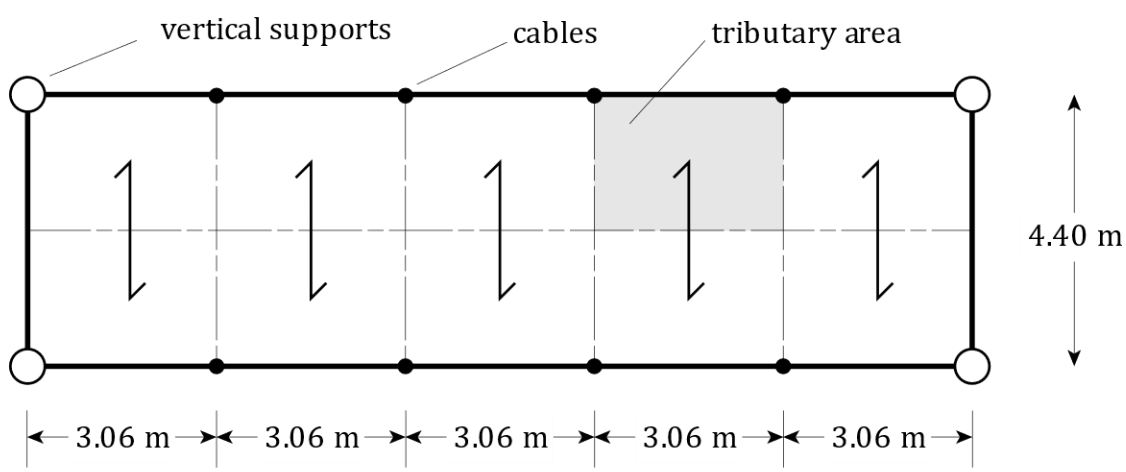


Figure A.1 Tributary area in typical bridge plan

2) Distributed load

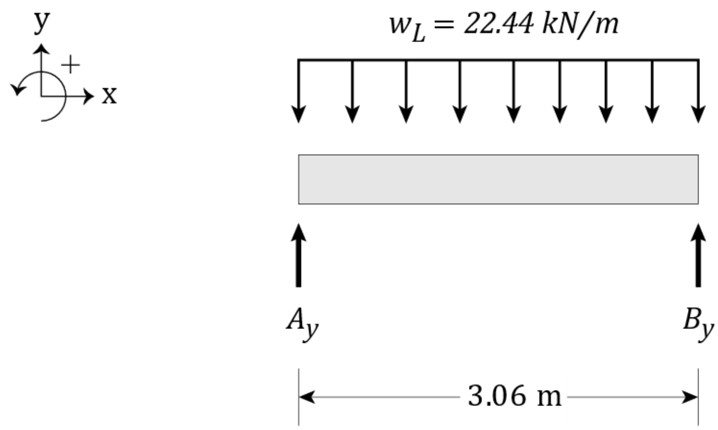
As seen in the bridge plan found in Appendix B: Drawing Package, this flat bridge area will support a covered pedestrian pathway in the middle, and the edges will be left outside. Due to the parabolic shape of the covered walkway, snow will likely slide to the sides and accumulate on the edges. For this reason, we will assume for precaution that the entire area should be capable of supporting both snow and pedestrian traffic. We therefore assume a load of 10.2 kPa as per the client's requirements (Atkins, 2020).

We use this value to calculate the distributed load.

$$10.2\ kPa \times \frac{4.40\ m}{2} = 10.2\ kPa \times 2.2m = 22.44 \frac{kN}{m}$$

3) Cable supports

Using equilibrium equations to calculate supports.



$$\circlearrowleft^{+} \quad \sum M_A = 0 = (B_y \times 3.06m) + \left(-22.44\frac{kN}{m} \times 3.06m \times \frac{3.06m}{2}\right)$$

$$B_y = 34.33\ kN \uparrow$$

$$\uparrow^{+} \quad \sum F_y = 0 = \left(-19.8\frac{kN}{m} \times 3.06m\right) + (A_y) + (15.147kN)$$

$$A_y = 34.33\ kN \uparrow$$

4) Total Load

Each cable is located where two supports of adjacent tributary areas meet. Thus, the total vertical load supported by each cable will be

$$C_y = 2 \times 34.33\ kN = 68.66\ kN$$

Now that the loads supported by the cables are known, we begin determining the funicular shape of the arch.

5) Addressing the diagonal cables

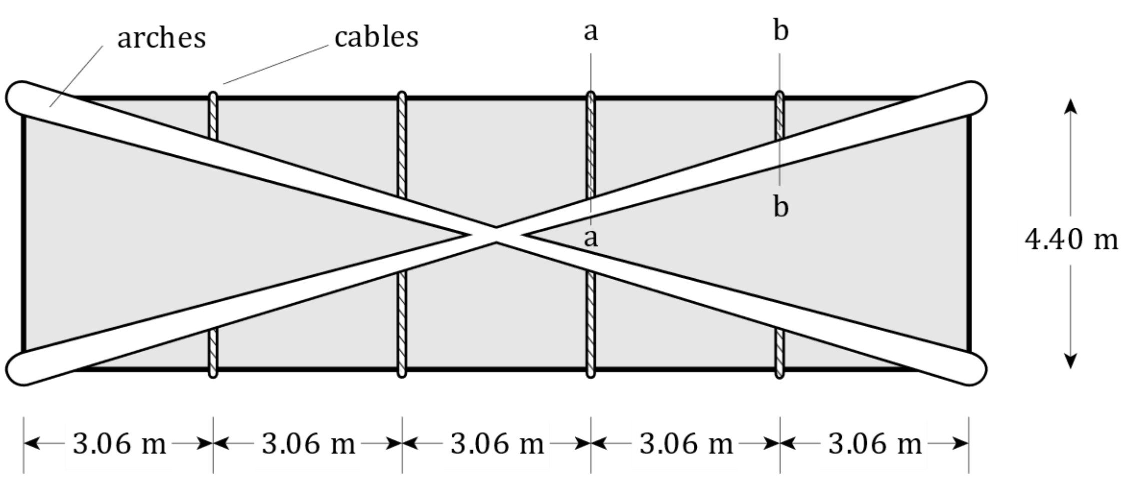


Figure A.2 Initial typical bridge plan

In order to allow space underneath the arches for a pedestrian walkway, the cables could not be installed vertically directly under their connection points, as seen in conventional arch bridges. Instead, the base of the cables are attached to the edges of the bridge as illustrated in Figure A.2 and meet the arches on a diagonal. This is better understood in a cross-section a-a and b-b illustrated in Figure A.3.

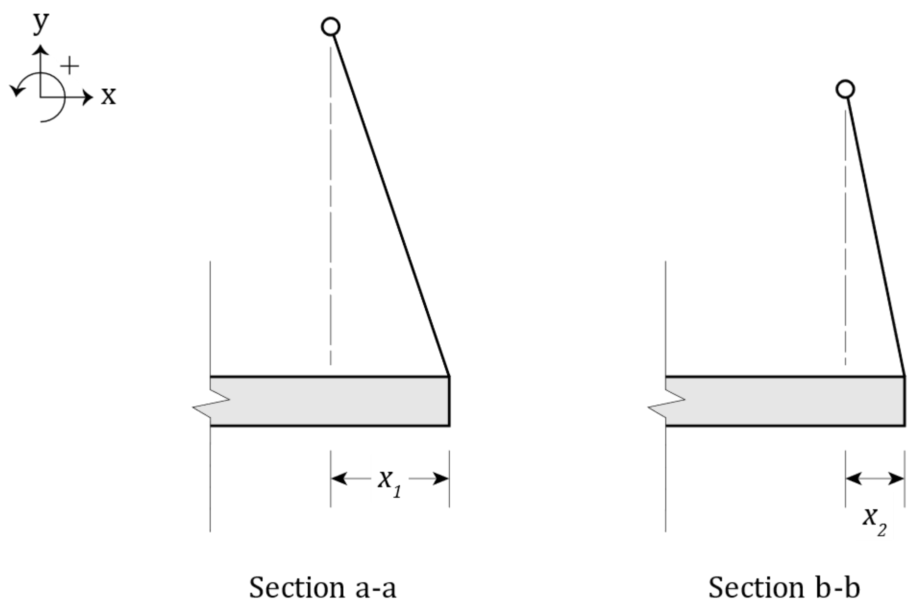
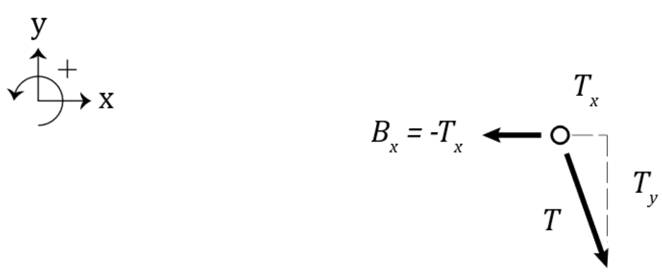


Figure A.3 Section cuts a-a and b-b

Due to the diagonal tension of the cables, the arches cross sections as observed in Figure A.3 will be subject to the y-component of the cable tension (calculated earlier as the vertical cable load) and a horizontal x-component as well. This additional force component will add significant complexity to the calculations, requiring us to go beyond the scope of AE205. To avoid this, we add bracing in the x-direction on the arch at the location of the cable connections. This will resist the horizontal component of the cables and allow us to assume each point on the arch at cable attachments will only be subject to the vertical component of the cable tension.



FBD of arch at cut a-a

$$\xrightarrow{+} \quad \sum F_x = T_x + (-T_x) = 0$$
$$\uparrow^{+} \quad \sum F_y = T_y = 68.66\ kN$$

*Note: The above FBD assumes the braces are sized large enough to resist the horizontal force component of the cable, T_x.

See Figure A.4 below for the updated typical bridge plan.

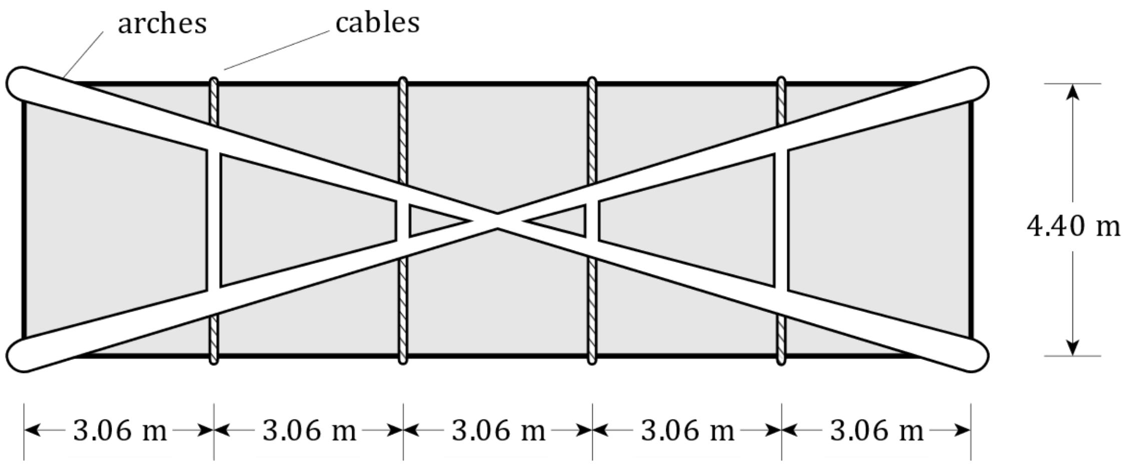
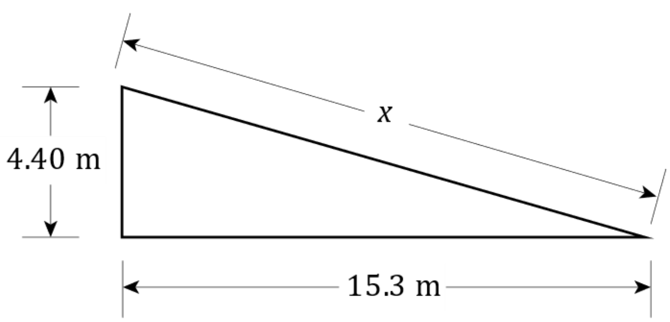


Figure A.4 Typical bridge plan with bracing

6) Changing the reference plane

In order to simply future calculation, we change the reference plane and draw a free-body diagram of one of the arches looking at it perpendicularly. To determine the spacing of the cable connections along the arch length, we calculate the total diagonal length of the arch using the Pythagorean theorem and divide by 5.



$$x^2 = (4.40)^2 + (15.3)^2$$
$$x = \sqrt{19.36 + 234.09}$$
$$x = 15.92\ m$$

$$\therefore connection\ spacing = \frac{15.92}{5} = 3.18\ m$$

Using this information, we can draw the free-body diagram of the arch in the new reference plane c-c indicated in Figure A.5.

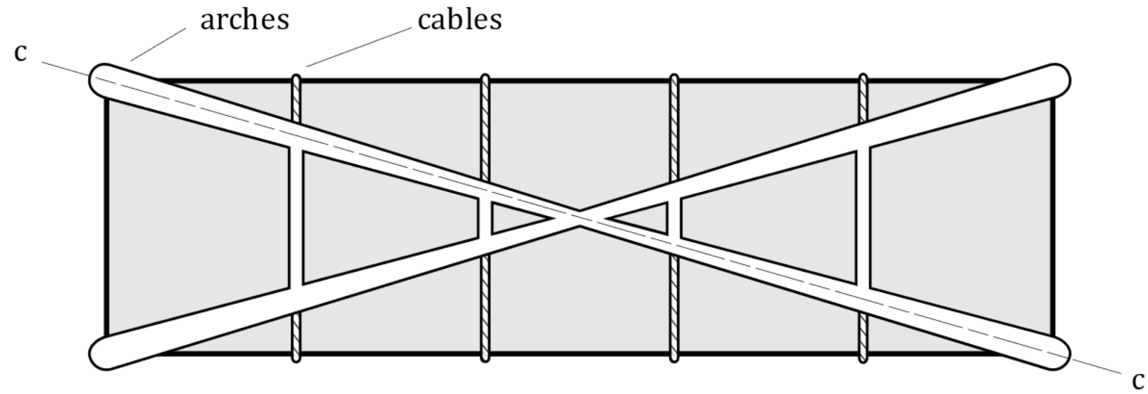


Figure A.5 New reference plane on typical bridge plan

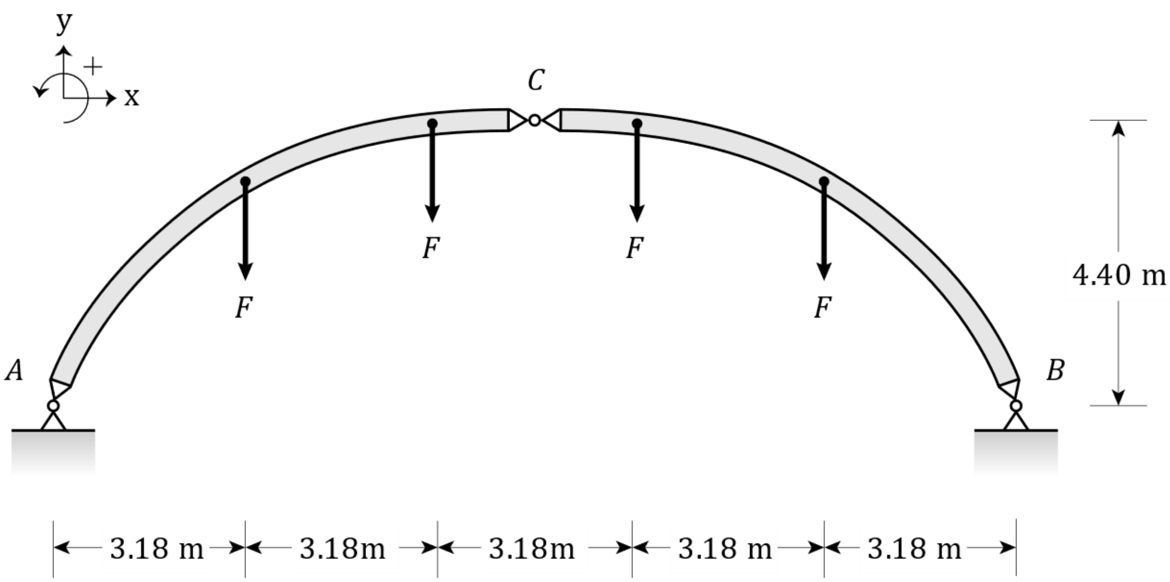


Figure A.6 FBD of an arch on plane c-c

7) General Cable Theorem

Before using the general cable theorem, we must acknowledge that a funicular arch subject to point loads – such as this one – will have linear members between the point loads. Note that in our final bridge, the curved arches are accomplished by placing ornamental cover pieces over the straight members, effectively concealing the structural system.

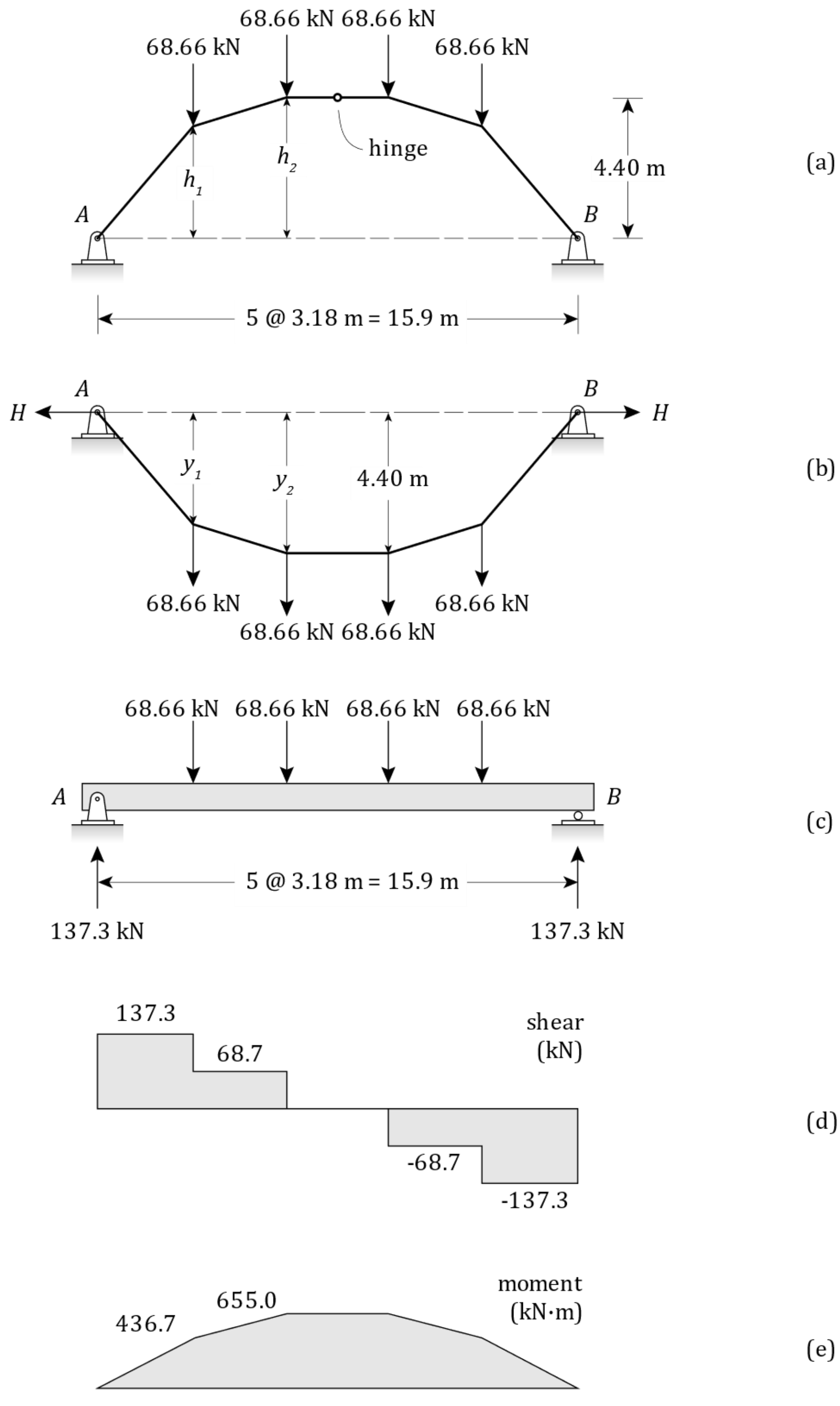


Figure A.7 Applying cable theory to establish funicular shape of arch

First, the funicular shape of the arch is established for our particular loading scenario in Figure A.7a. The rise of the arch at midspan is set at 4.40m.

We imagine that the set of loads is applied to a cable that spans the same distance as the arch (Figure A.7b). The sag of the cable is set as 4.40m, the same height of the arch at midspan. Applying the general cable theory, we imagine that the loads supported by the cables are applied to an imaginary simply supported beam with a span equal to that of the cable (Figure A.7c). We calculate its support reactions.

$$\uparrow^{+} \quad \sum F_y = 0$$

$$A_y = B_y = \frac{4 \times 68.66\ kN}{2} = 137.32\ kN \uparrow$$

We next construct the shear and moment curves. The shear values are found in a straightforward manner, following the direction of the forces on the beam from left to right.

$$V_1 = 137.32\ kN \quad V_2 = 68.66\ kN \quad V_3 = 0 \quad V_4 = -68.66\ kN \quad V_5 = 137.32\ kN$$

Noticing the symmetry, only two values of the moment curve are calculated.

$$M_1 = 137.32\ kN \times 3.18m = 436.67\ kN \cdot m$$
$$M_2 = 68.66\ kN \times 3.18m + 436.67\ kN \cdot m = 655.01\ kN \cdot m$$

According to the general cable theorem (Leet, Uang, Lanning, & Gilbert, 2018) at every point,

$$M = H \cdot h_z \qquad \text{(A.1)}$$

where M = moment at an arbitrary point in the beam
H = horizontal component of support reaction
h_z = cable sag at an arbitrary point

Since h = 4.40 m at midspan and M = 655.01 kN·m, we apply Equation A.1 at that point to find H.

$$H = \frac{M}{h} = \frac{655.01}{4.40} = 148.87\ kN$$

With H established, we apply Equation A.1 at 3.18m from A to compute h_1.

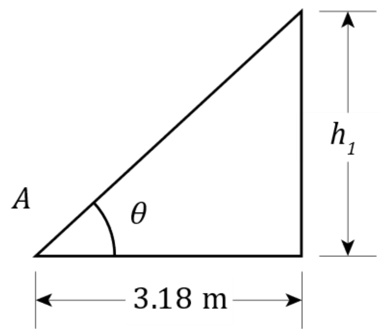
$$h_1 = \frac{M}{H} = \frac{436.67}{148.87} = 2.93\ m$$

We also note that h_2 is at the same height as the midspan, and thus:

$$h_2 = 4.40\ m$$

8) Max Compressive stress in arch

Max compression in funicular arches (T_f) occur at the steepest slope, in this case at the supports.



Triangle formed at support A with h_1

$$\tan\theta = \frac{h_1}{3.18\,m} = \frac{2.93\,m}{3.18\,m} = 0.9214$$
$$\theta = \tan^{-1} 0.9214 = 42.66°$$

$$T_f = \frac{H}{\cos 42.66°} = \frac{148.87}{\cos 42.66°} = 202.44\,kN$$

9) Parabolic Approximation

In order to accomplish the curved aesthetic of the arches, a smooth cover will be placed around the straight members of the funicular structure we solved earlier. To help reduce material waste, we must find the curve that fits most closely our structure, creating the tightest fit possible.

For the sake of simplicity, we will assume this curve is parabolic, and thus will be defined by the following formula.

$$y = a(x-h)^2 + k \qquad \text{(A.2)}$$

We let the bottom left corner of the arch represent the origin out our coordinate system, making the vertex coordinate (7.96, 4.40) m. We sub in these known points in Equation A.2 and solve for the remaining unknown.

$$y = a(x-7.96)^2 + 4.4$$
$$y(0) = 0 = a(-7.96)^2 + 4.4$$
$$a = -\frac{4.4}{(-7.96)^2}$$
$$a = 0.070$$

$$y = -0.07(x-7.96)^2 + 4.4$$

10) Sizing the arch

Since the arch will be primarily subject to compression given its funicular geometry, the most important failure to avoid is buckling. Thus, this is the failure mode we will use to size the arch. Euler's column formula described below will be used.

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} \quad \text{(A.3)}$$

where P_{cr} = critical force at buckling
E = modulus of elasticity
I = moment of inertia
k = effective length factor
L = column length

Looking at member AB in Figure A.8, we notice an effective length of 0.7 due to pin (support) and fixed (weld) connections, and the max compressive force T_f going through it. This is illustrated as a standalone free-body diagram in Figure A.9a. Since all other members are made of the same material, are subject to less load, are shorter, and have equal or smaller effective length factors, we know from Equation A.3 that this member will buckle at the lowest critical load. Thus, we use this first arch section for the sizing of all arch members.

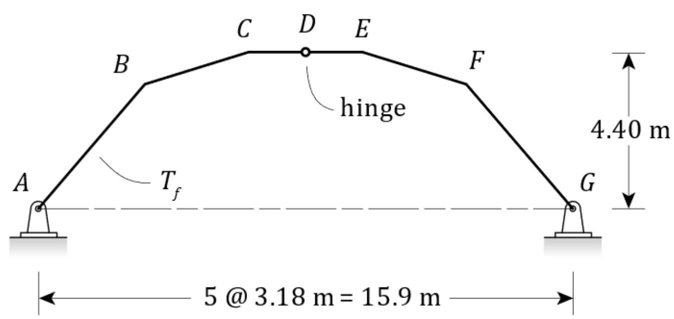


Figure A.8 Arch diagram with labelled joints and location of T_f

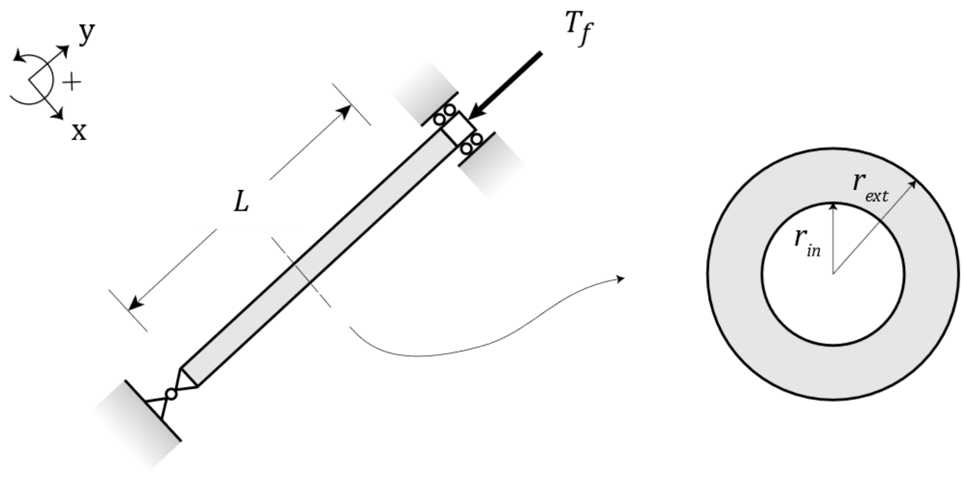


Figure A.9 FBD of first straight

In order to account for extreme loading scenarios and imperfections in materials and construction, we implement a factor of safety of 1.5 on all critical loads.

$$T_f = 202.44\ kN$$

$$P_{cr} = T_f \times F.S.$$
$$= 202.44 \times 1.5$$
$$P_{cr} = 303.66\ kN$$

$$L = \sqrt{(3.18)^2 + (2.93)^2}$$
$$L = 4.324\ m$$

$$k = 0.7$$

$$E = 200\ GPa$$

Given these values and Equation A.3, we find the required moment of inertia for the arch cross section.

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2}$$
$$303.66\ kN = \frac{\pi^2 (200\ GPa) I}{(0.7 \times 4.324m)^2}$$
$$I = 1.4094 \times 10^6\ mm^4$$

Since the structural members in the arches will be subject to mostly compressive forces, are curved, and must be relatively light-weight to minimize dead-load, steel HSS columns were selected as the optimal structural material. Since an aesthetic cover will be added around the steel, we wanted the cross section to have a smaller perimeter, to save on material cost. This informed the choice of a round cross section for the HSS column, illustrated in Figure A.9b.

The Steel Tube Institute resources were used to find a round HSS steel member capable of generating the required moment of inertia. Properties of the selected steel member is tabulated below (Steel Tube Institute, 2020).

Name, imperial	r_{ext}, mm	Thickness, mm	Area, mm^2	I, mm^4
HSS3.5X0.25	88.9	6.35	1645.16	1.411×10^6

Table A.1 Round HSS column properties

11) Checking yielding stress

As an additional measure of precaution, we now check if this cross-section is at risk of yielding. We do so by calculating the axial force required to cause steel to yield, which occurs at 350 MPa (Atkins, 2020).

$$\sigma_y = \frac{P}{A}$$

$$350\ MPa = \frac{P}{1645.16\ mm^2}$$

$$P = 575.8\ kN$$

Since $P > T_f$, the structural member will never experience the force required to cause it to yield.

12) Weight reduction

To appreciate the savings in material and weight associated with the choice of a hollow structural section, we can use to Equation A.3 to calculate the required area of a round section made of solid steel. This results in an area of 4208.41 mm^2. Since area is proportional to weight assuming constant cross-section, we can calculate the weight reduction as follows.

$$weight\ reduction = \frac{A_{hollow}}{A_{solid}} \times 100\% = \frac{1645.16}{4208.41} \times 100\% = 39\%$$

Thus, by choosing a hollow structural section over solid steel, we save around 39% in cost and weight.

Cables

1) Computing max tension

With the heights of the cable attachments calculated in Arches step 7, all that is left to define the geometry of the cables is the depth of their connections, labelled x_1 and x_2 in Figure A.10.

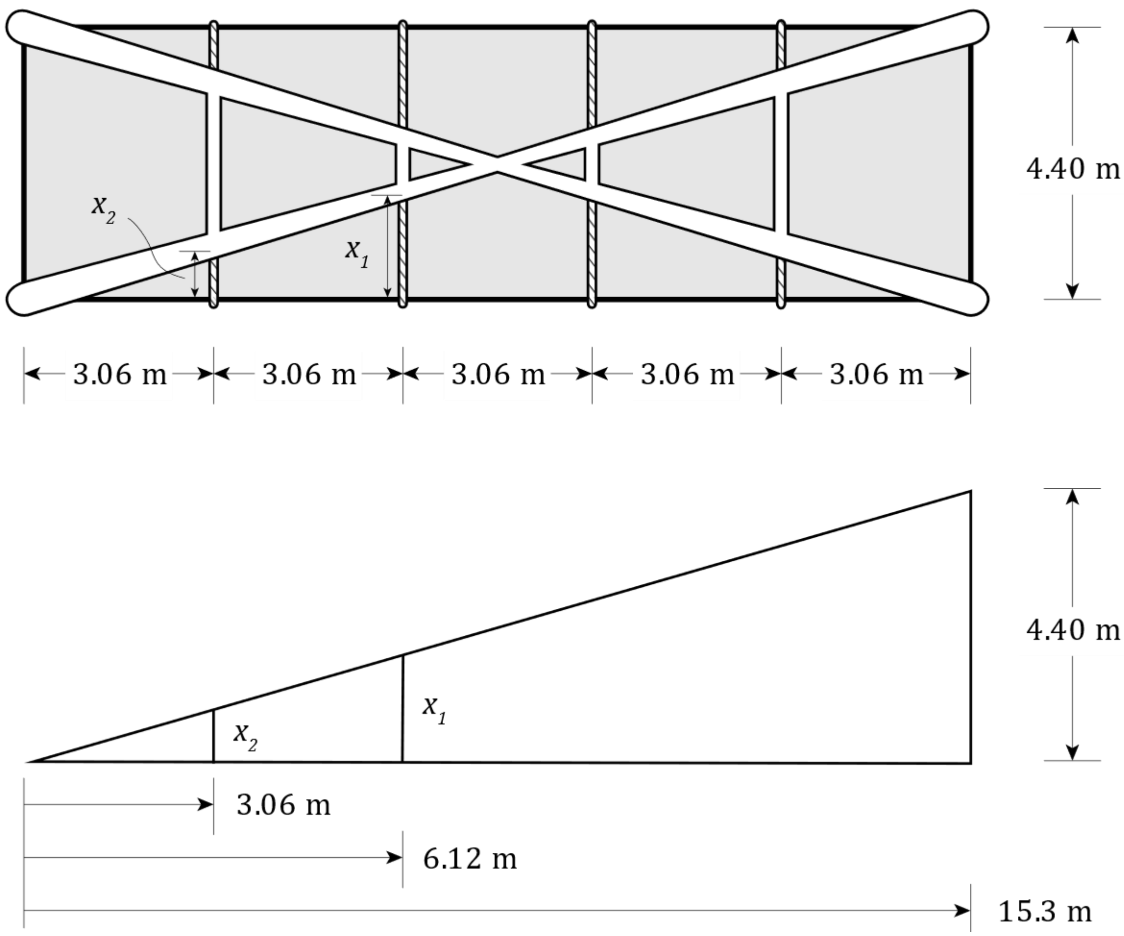


Figure A.10 Similar triangle for cable connection depths

Using similar triangles:

$$\frac{x_1}{6.12\ m} = \frac{4.40\ m}{15.3\ m} \qquad x_1 = 1.76\ m$$

$$\frac{x_2}{3.06\ m} = \frac{4.40\ m}{15.3\ m} \qquad x_2 = 0.88\ m$$

Combining this information with the vertical force calculated in Arches 4), and the height of the cable connection established in Arches 7), trigonometry is used to calculate the max tension in the cables. Cuts a-a and b-b from Figure A.3 are re-used to display dimensions.

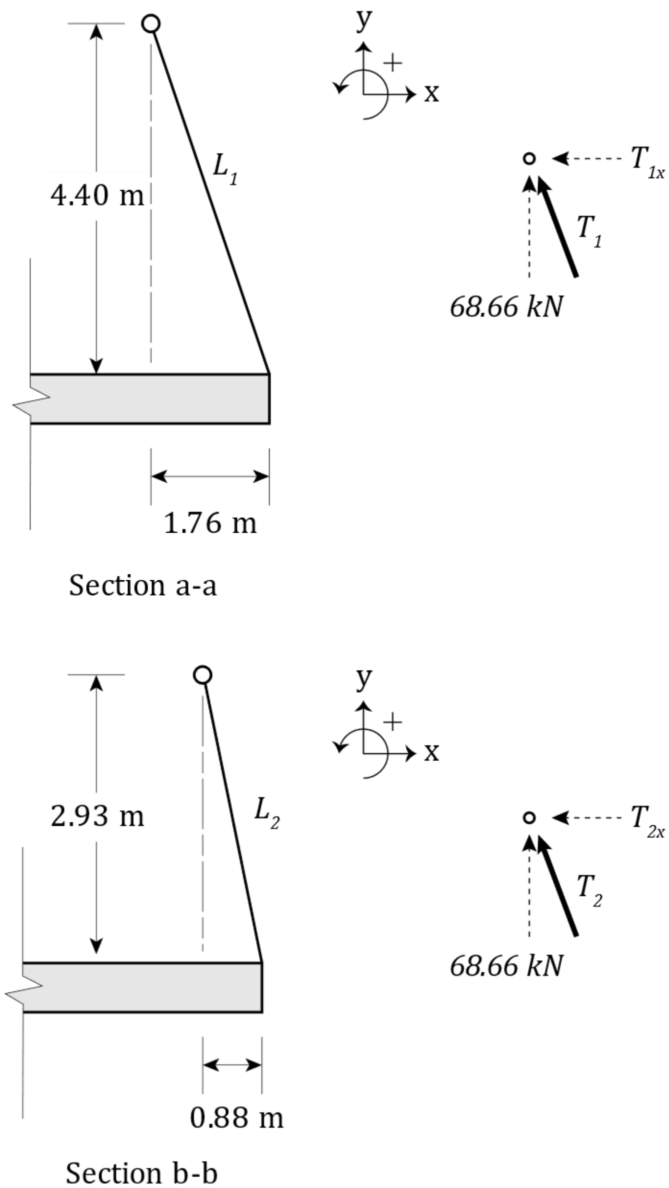


Figure A.11 Section cuts and their respective FBD diagrams

Calculating max tension.

$$L_1 = \sqrt{4.4^2 + 1.76^2}$$
$$L_1 = 4.739\ m$$

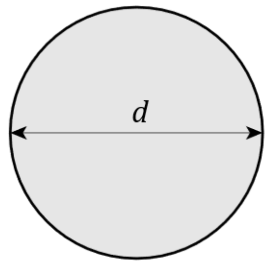
$$\frac{T_1}{68.66\ kN} = \frac{4.739\ m}{4.40\ m}$$
$$T_1 = 73.95\ kN$$

$$L_2 = \sqrt{4.4^4 + 0.88^2}$$
$$L_2 = 4.487\ m$$

$$\frac{T_2}{68.66\ kN} = \frac{4.487\ m}{4.40\ m}$$
$$T_1 = 70.02\ kN$$

2) Sizing cables

From client requirements, the cables will be made of high strength steel with an elastic modulus of 200 GPa, and a yield stress of 600 MPa. For safety, both sets of cables will be sized using the largest load between the two. In addition, a factor of safety of 1.5 will be applied to the greater tension value.



Steel cable cross-section

$$T_2 > T_1$$
$$\therefore T_{max} = T_2 = 73.95\ kN$$

$$P = T_{max} \times F.S. = 73.95\ kN \times 1.5$$
$$P = 110.925\ kN$$

$$\sigma_Y = \frac{P}{A}$$
$$600\ MPa = \frac{110.925\ kN}{A}$$
$$A = 184.875\ mm^2$$

$$A = 184.875\ mm^2 = \frac{\pi d^2}{4}$$
$$d = 15.3\ mm$$

Thus, all cables with have a diameter of size 15.3 mm.

Beams

1) Tributary area

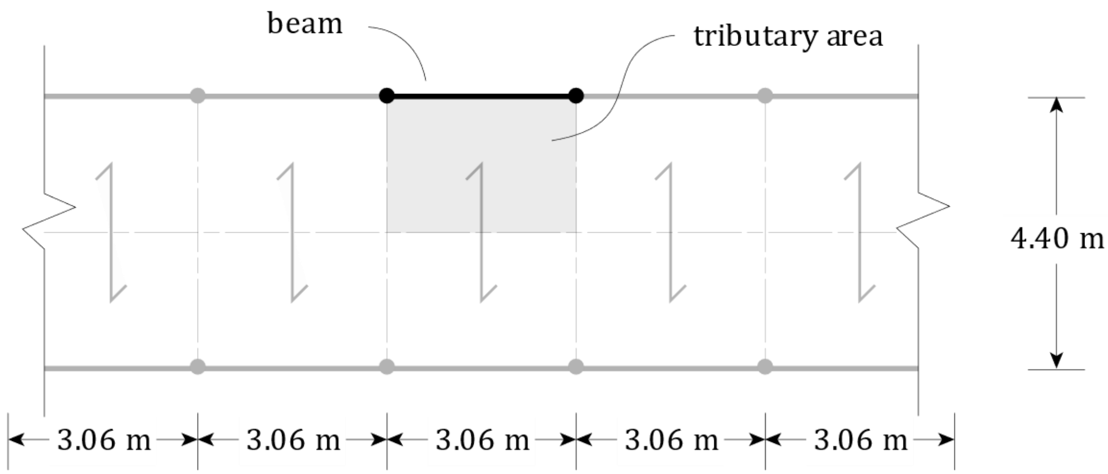


Figure A.12 Tributary area and its associated beam

From the Arches calculations step 2, we know that the distributed load associated with the tributary area illustrated in Figure A.12 is 22.44 kN/m.

2) Normal, shear and moment diagrams

To simplify calculations, we will assume that the beam highlighted in Figure A.12 is supported by pin connection on both ends. Consequentially, the deflection measured in future step might be an overestimate, which is a favorable outcome for safety.

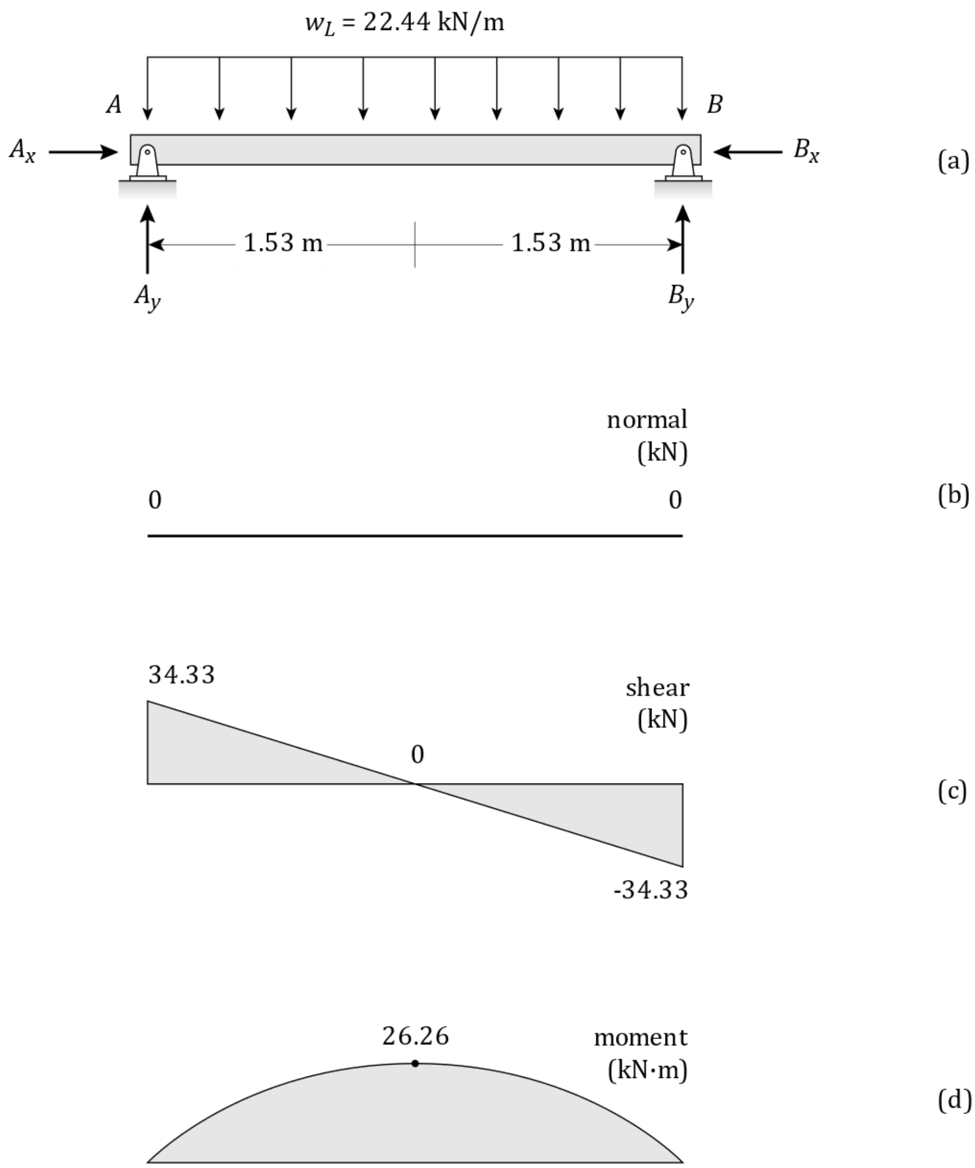


Figure A.13 FBD, normal, shear, and moment diagrams

Starting with the supports, we assume negligible loads along the x direction.

3) Deflection at beam mid-span

The following calculation utilizes the principle of virtual work to calculate the deflection in beam AB shown in Figure A.13a.

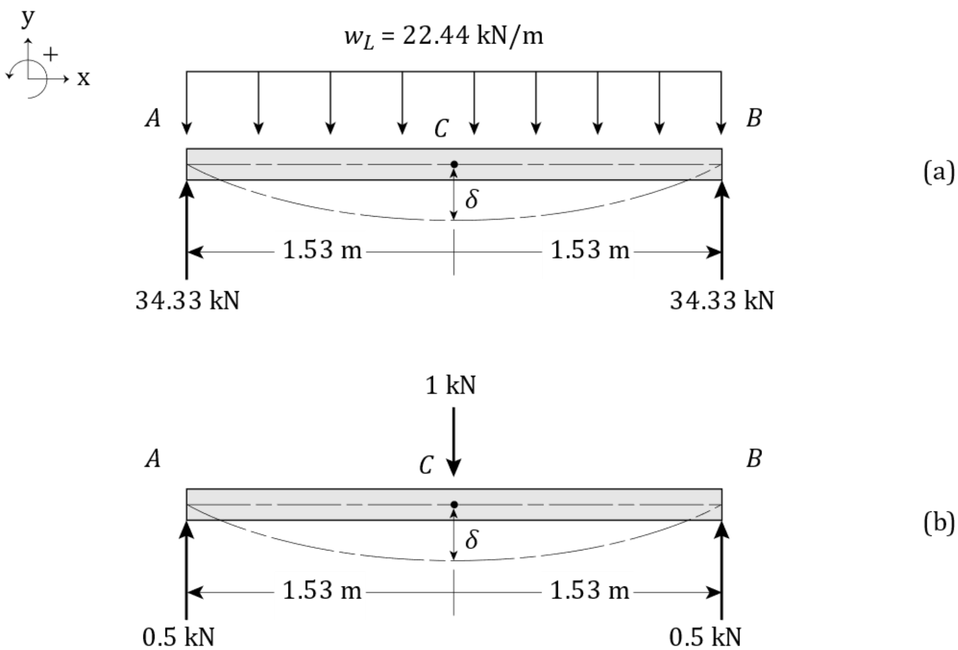
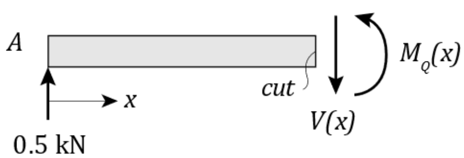


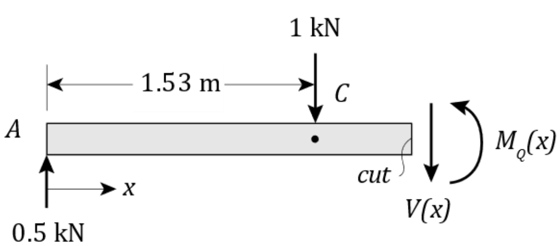
Figure A.14 P-system (a) and Q-system (b) for displacement

Finding equations for M_Q using distance x:



From 0 to 1.53 m

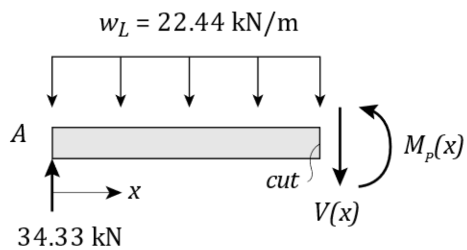
$$\circlearrowleft^{+} \quad \sum M_{cut} = 0 = -0.5(x) + M_Q(x)$$
$$M_Q(x) = 0.5x$$



From 1.53 m to 3.06 m

$$\circlearrowleft^{+} \quad \sum M_{cut} = 0 = -0.5(x) + M_Q(x) + 1(x - 1.53)$$
$$M_Q(x) = 0.5x$$

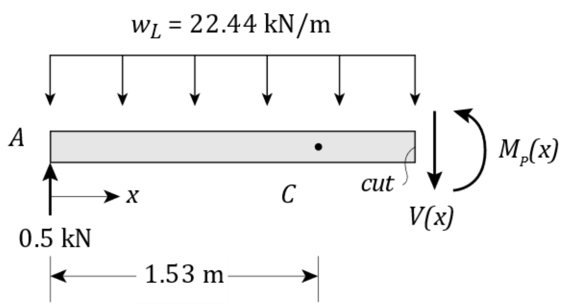
Finding equations for M_P using distance x:



From 0 to 1.53 m

$$\circlearrowleft^{+} \quad \sum M_{cut} = 0 = \left(22.44\frac{kN}{m} \cdot x \cdot \frac{x}{2}\right) + (-34.33\ kN \cdot x) + M_P(x)$$

$$M_P(x) = 34.33x - 11.22x^2$$



From 1.53 m to 3.06 m

$$\circlearrowleft^{+} \quad \sum M_{cut} = 0 = \left(22.44\frac{kN}{m} \cdot x \cdot \frac{x}{2}\right) + (-34.33\ kN \cdot x) + M_P(x)$$

$$M_P(x) = 34.33x - 11.22x^2$$

Using the principal of virtual work and subbing in values of M_Q and M_P:

$$Q \cdot \delta_C = \sum \int_0^L \frac{M_Q \cdot M_P}{EI} dx$$

$$Q \cdot \delta_C = \int_0^{1.53} \frac{(0.5x)(34.333x - 11.22x^2)}{EI} dx$$

$$+ \int_{1.53}^{3.06} \frac{(1.53 - 0.5x)(34.333x - 11.22x^2)}{EI} dx$$

$$Q \cdot \delta_C = \frac{1}{EI}\left[\int_0^{1.53} 17.167x^2 - 5.61x^3\, dx\right]$$

$$+\frac{1}{EI}\left[\int_{1.53}^{3.06} 52.529x - 17.167x^2 - 17.167x^2 + 5.61x^3\, dx\right]$$

$$Q \cdot \delta_C = \frac{1}{EI}[5.722x^3 - 1.403x^4]_0^{1.53} + \frac{1}{EI}[26.265x^2 - 11.444x^3 + 1.403x^4]_{1.53}^{3.06}$$

$$Q \cdot \delta_C = \frac{1}{EI}[12.809]$$

$$+\frac{1}{EI}[(245.93 - 327.90) + 122.967) - (61.483 - 40.988 + 7.685)]$$

$$(1) \cdot \delta_C = \frac{1}{EI}[12.809] + \frac{1}{EI}[12.817]$$

$$\delta_C = \frac{25.626\ kN \cdot m^3}{EI_{beam}}$$

We allow a deflection 300th of the length of the beam span.

$$\delta_C = \frac{span}{300}$$

$$\delta_C = \frac{3060mm}{300} = 10.2mm$$

Using this deflection value in out PVW equation, we can solve for the beam's required moment of inertia.

$$(10.2mm) = \frac{25.626\ kN \cdot m^3}{(200{,}000\ \frac{N}{mm^2})(I_{beam})}$$

$$I_{beam} = \frac{2.5626x10^{13}\ N \cdot mm^3}{(200{,}000\ \frac{N}{mm^2})(10.2mm)}$$

$$I_{beam} = 1.256x10^7 mm^4$$

Implementing a factor of safety of 1.5 we get

$$I_{beam} = 1884\ cm^4$$

The Engineering Toolbox was used to find a wide-flange steel beam capable of generating the required moment of inertia. Properties of the selected steel member is tabulated below (Engineering Toolbox, 2008).

Name, imperial	Width, mm	Height, mm	Thickness, mm	Area, cm^2	I, cm^4
W8X4X15	206	102	6.2	28.6	2004

Table A.2 Wide-flange steel beam properties

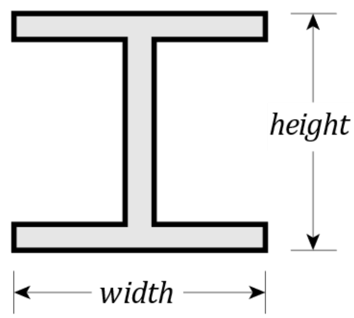


Figure A. 15 I beam dimensions

4) Rotation at one end of the beam

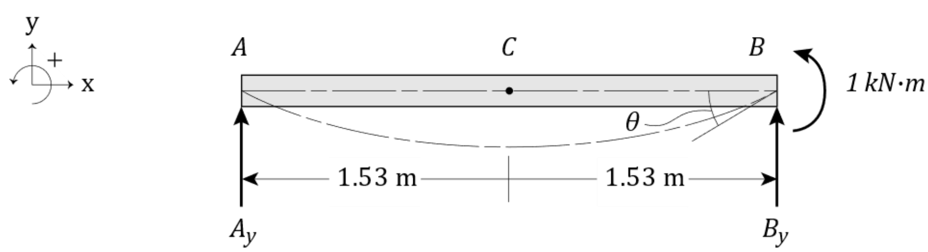


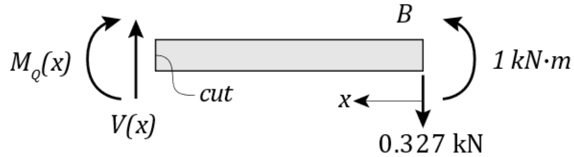
Figure A.16 Q-system for rotation

Calculating supports.

$$\circlearrowleft^{+} \quad \sum M_A = 0 = 1\ kN \cdot m + (B_y \times 3.06\ m)$$
$$B_y = -0.327\ kN = 0.327\ kN\ \downarrow$$

$$\uparrow^{+} \quad \sum F_y = 0 = -0.327\ kN + A_y$$
$$A_y = 0.327\ kN\ \uparrow$$

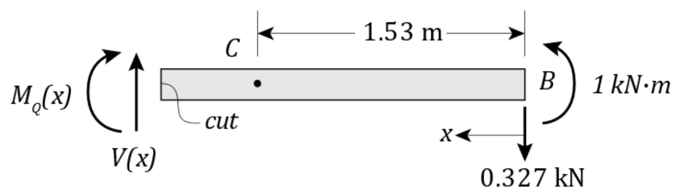
Finding equations for M_Q using distance x:



From B, 0 to 1.53 m

$\circlearrowleft^{+}$ $\sum M_{cut} = 0 = 1\,kN \cdot m - M_Q(x) + (-0.327x)$

$M_Q(x) = 1 - 0.327x$

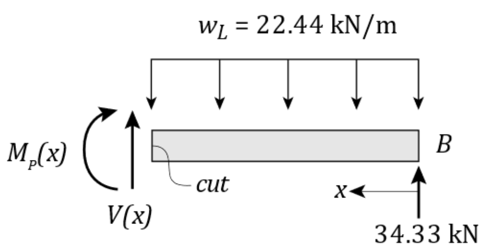


From B, 1.53 m to 3.06 m

$\circlearrowleft^{+}$ $\sum M_{cut} = 0 = 1\,kN \cdot m - M_Q(x) + (-0.327x)$

$M_Q(x) = 1 - 0.327x$

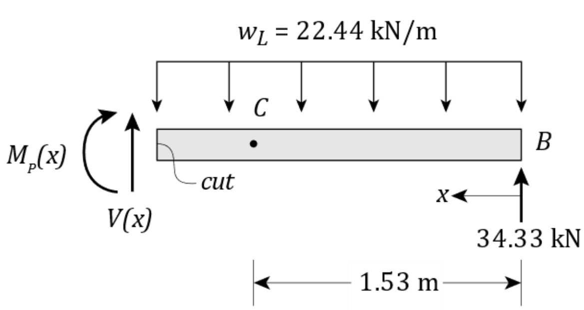
Finding equations for M_P using distance x:



From B, 0 to 1.53 m

$\circlearrowleft^{+}$ $\sum M_{cut} = 0 = \left(-22.44\frac{kN}{m} \cdot x \cdot \frac{x}{2}\right) + (34.33\,kN \cdot x) - M_P(x)$

$M_P(x) = 34.33x - 11.22x^2$



From B, 1.53 m to 3.06 m

$$\circlearrowleft^{+} \quad \sum M_{cut} = 0 = \left(-22.44\frac{kN}{m}\cdot x\cdot\frac{x}{2}\right) + (34.33\ kN\cdot x) - M_P(x)$$

$$M_P(x) = 34.33x - 11.22x^2$$

Using the principal of virtual work and subbing in values of M_Q and M_P:

$$Q\cdot\theta_P = \sum\int_0^L \frac{M_Q\cdot M_P}{EI}dx$$

$$Q\cdot\theta_P = \int_0^{1.53}\frac{(1-0.3268x)(34.333x-11.22x^2)}{EI}dx$$

$$+\int_{1.53}^{3.06}\frac{(1-0.3268x)(34.333x-11.22x^2)}{EI}dx$$

$$Q\cdot\theta_P = \frac{1}{EI}\left[\int_0^{1.53} 34.333x - 11.22x^2 - 11.22x^2 + 3.667x^3\ dx\right]$$

$$+\frac{1}{EI}\left[\int_{1.53}^{3.06} 34.333x - 11.22x^2 - 11.22x^2 + 3.667x^3\ dx\right]$$

$$Q\cdot\theta_P = \frac{1}{EI}[17.167x^2 - 7.48x^3 + 0.917x^4]_0^{1.53}$$

$$+\frac{1}{EI}[17.167x^2 - 7.48x^3 + 0.917x^4]_{1.53}^{3.06}$$

$$Q\cdot\theta_P = \frac{1}{EI}[18.418] + \frac{1}{EI}[26.801 - 18.418]$$

$$(1\ KN)\cdot\theta_P = \frac{26.800\ KN^2\cdot m^3}{(200\ \frac{kN}{mm^2})(2.004x10^7mm^4)}\cdot\frac{1000}{(1x10^9)(1x10^{-3})^4}$$

$$\theta_P = 6.687x10^{-3}\ rad$$

5) Planar state of stress

As requested by the client, planar state stress will be evaluated at 100 mm below the neutral axis. We inspect the internal forces in the beam by taking a cut at midspan, illustrated in Figure A.17.

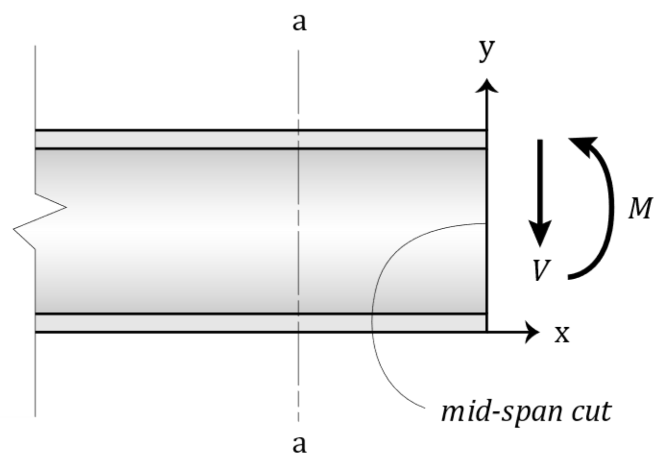


Figure A.17 Internal forces of I-beam at mid-span

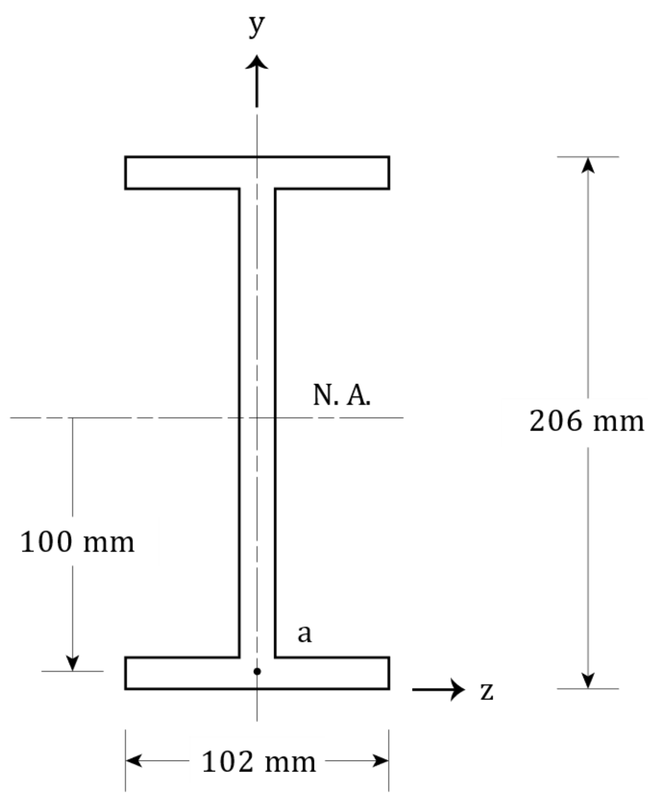


Figure A. 18 I-beam cross-section

Section Properties

M_{mid} = 26.27 kN·m
V_{mid} = 0 kN
c = 100 mm
E = 200 GPa

Calculating bending stress.

$$Bending\ Stress = \frac{Mc}{I}$$

$$\sigma_B = \frac{(26264700\ N{\cdot}mm)(100mm)}{2.004x10^7 mm^4}$$

$$\sigma_B = 131.06\ MPa$$

A positive moment will generate tension at the bottom of the beam.

Calculating shear stress.

$$Shear\ Stress = \frac{VQ}{It}$$

As observed in Figure A.11c, the beam experiences no shear at its mid-span. Thus, shear stress will be 0.

$$\tau_{xy} = 0\ kN$$

Summarizing stresses orthogonal to frame of reference.

$$\tau_{xy} = \tau_{xz} = \tau_{zy} = 0\ kN$$

$$\sigma_y = \sigma_z = 0\ kN$$

$$\sigma_x = 131.06\ MPa$$

Calculating the principle stresses at the mid-span of the beam.

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_1 = \frac{(131.06) + (0)}{2} + \sqrt{\left(\frac{131.06 - 0}{2}\right)^2 + (0)^2}$$

$$\sigma_1 = 131.06\ MPa$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_2 = \frac{(131.06) + (0)}{2} + \sqrt{\left(\frac{131.06 - 0}{2}\right)^2 + (0)^2}$$

$$\sigma_2 = 0\ MPa$$

6) Maximum in-plane / out-plane stresses

$$\tau_{max\ in-plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\tau_{max\ in-plane} = \sqrt{\left(\frac{131.06 - 0}{2}\right)^2 + (0)^2}$$

$$\tau_{max\ in-plane} = 65.53\ MPa$$

$$\tau_{max\ out-plane} = -65.53\ MPa$$

7) Von mises theory of failure

$$\sigma_y{}^2 \geq \sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2$$

Where,

$\sigma_1 = 131.06\ MPa$

$\sigma_2 = 0\ MPa$

$\sigma_y = 350\ MPa$

$$(350)^2 \geq (131.06)^2 + (0)^2 - (131.06)(0)$$

$$122500\ MPa^2 \geq 17176.72\ MPa^2$$

Therefore, yielding will not occur.

Piers

1) Pier design

How the load will travel:

i. Cables hold up the weight (UDL) generated on the deck
ii. Arch carries this load and passes it along the parabolic length
iii. Beams at ends support the arches
iv. The piers will carry 2.5 beams on either side, therefore 5 beams in total get passed into the pier

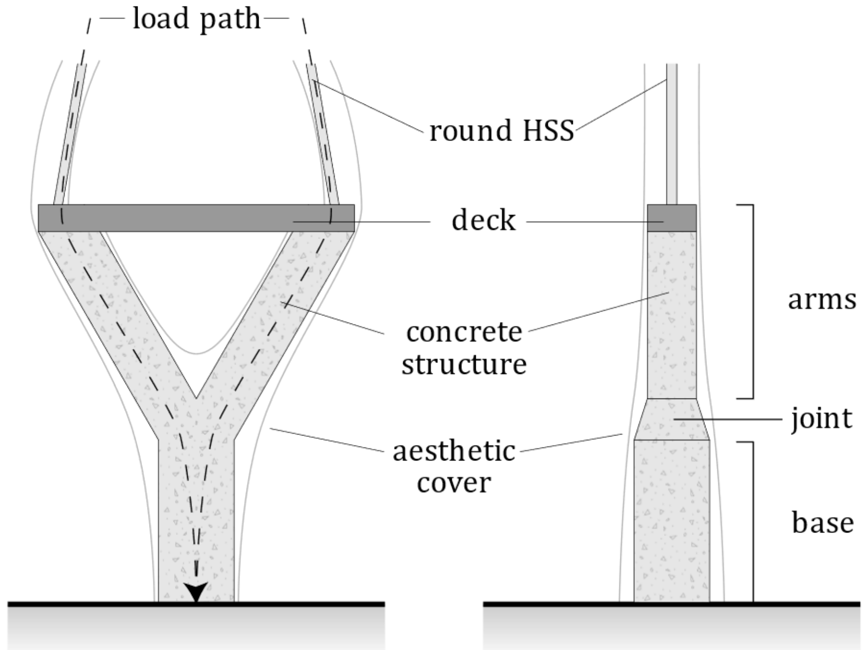


Figure A.19 Pier design diagrams

Arch supports (A_y) = 171.665 kN

Since there are two arches coming to one side of pier, therefore the beam across pier will double:

A_y = 343.333 kN ↑
B_y = 343.333 kN ↑

Therefore, the pier fork is simplified to a "Y" shape and will have a moulding around it to match the parabolic arch for aesthetic reasons.

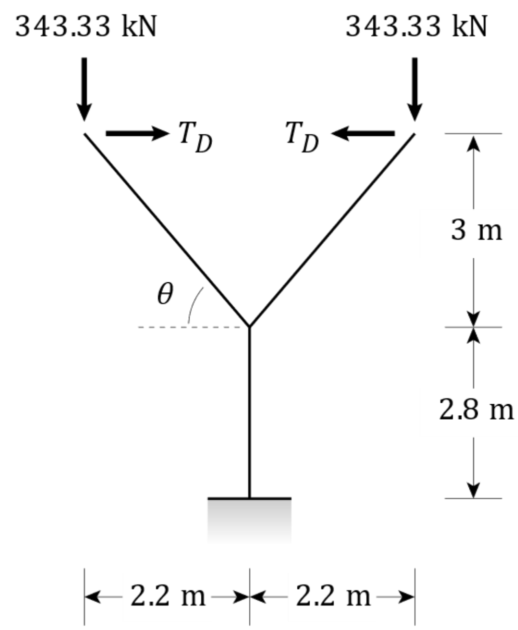


Figure A.20 External loads acting on piers

The pier will entirely made of Concrete

E = 17 GPa
f'c = 50 MPa

The cross-section will be square to facilitate construction.

2) Avoiding buckling in pier arms.

Calculating length of arm.

$$L = \sqrt{3^2 + 2.2^2}$$
$$L = 3.720m$$

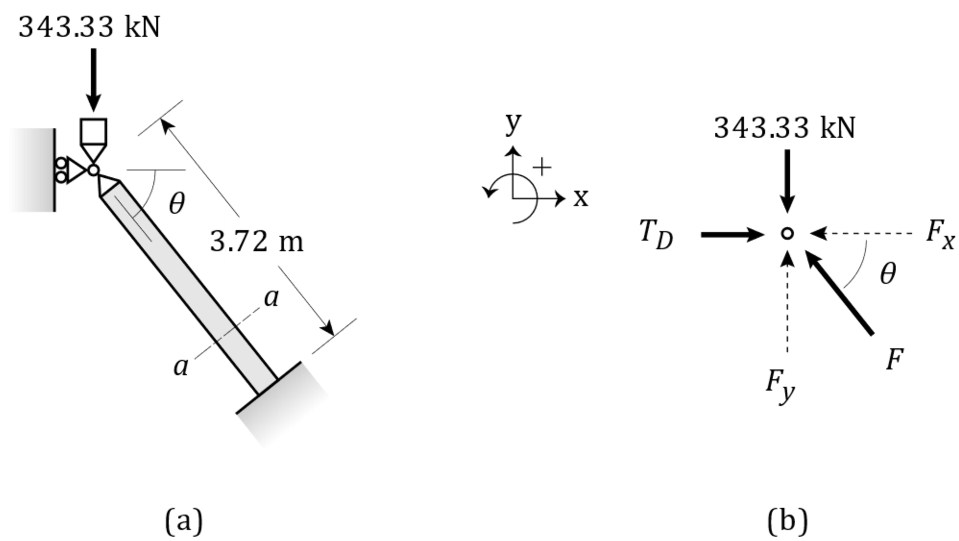


Figure A.21 Left-arm diagram (a) left-arm FBD (b)

Calculating angle theta.

$$\theta = \tan^{-1}\frac{3}{2.2}$$
$$\theta = 53.75°$$

Computing axial loading.

$$F_{Axial} = F = \frac{F_y}{\cos(90° - \theta)} = \frac{343.33\ kN}{\cos 36.25} = 425.733\ kN$$
$$F_x = F\cos\theta = (580.626\ kN)\cos 53.75° = 251.740\ kN$$

Given a pin-fixed connection, the effective length factor will be 0.7. Using this information and Equation A.3, we determine the minimum moment of inertia required to avoid buckling. A safety factor of 1.5 is applied first.

$$P_{cr} = F \times F.S.$$
$$P_{cr} = 425.733 \times 1.5$$
$$P_{cr} = 638.599\ kN$$

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2}$$
$$(638\,599\ N) = \frac{\pi^2(17\,000\ MPa)(I)}{(0.7 \cdot 3720mm)^2}$$
$$I = 2.581 \times 10^7 mm^4$$

With this moment of inertia, the column will only buckle at a critical load 1.5 times greater than the maximum axial load it should ever be subject to.

3) Avoiding buckling of pier base

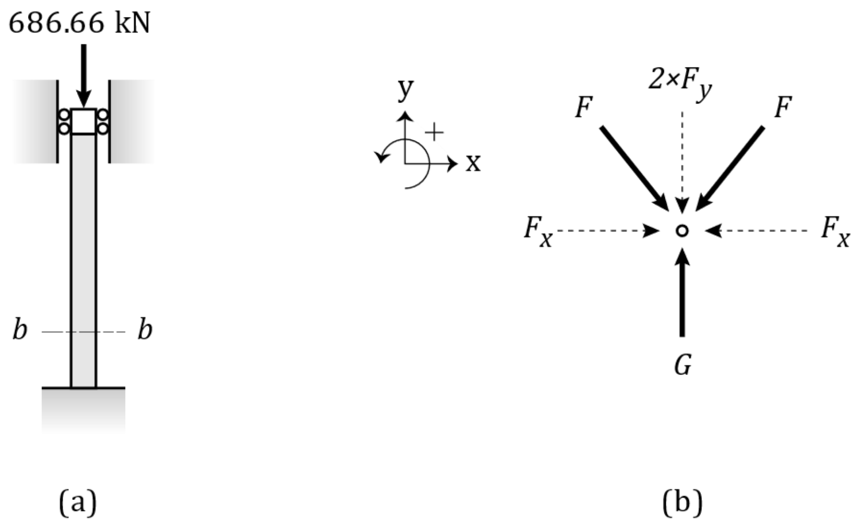


Figure A.22 Pier diagram (a) pier FBD (b)

Given a fixed-fixed connection, the effective length factor will be 0.5. Using this information and Equation A.3, we determine the minimum moment of inertia required to avoid buckling, A safety factor of 1.5 is applied first.

$$G = 343.33 \times 2$$
$$G = 686.66\ kN$$
$$P_{cr} = G \times F.S.$$
$$P_{cr} = 686.66\ kN \times 1.5$$
$$P_{cr} = 1029.99\ kN$$

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2}$$
$$(1\,029\,990\ N) = \frac{\pi^2(17\,000\ MPa)(I)}{(0.5 \times 2800mm)^2}$$
$$I = 1.203 \times 10^7 mm^4$$

With this moment of inertia, the column will only buckle at a critical load 1.5 times greater than the maximum axial load it should ever be subject to.

4) Sizing the structural members of the pier.

The a-a and b-b cuts from Figure A.21a and Figure A.22a respectively are illustrated below.

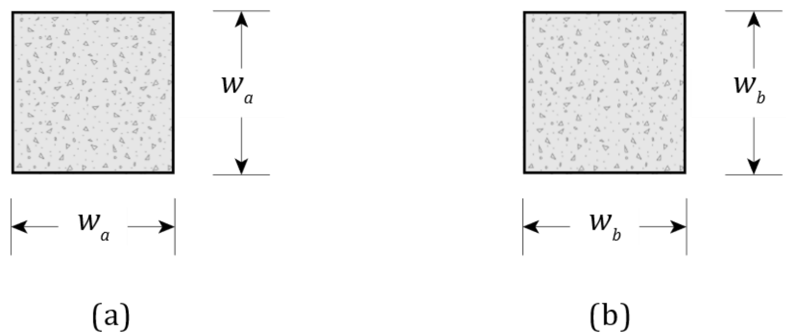


Figure A.23 Pier cross-section of the arms (a) and base (b)

Using the calculated moment of inertia, we size the arm and base of the pier.

$$I = \frac{w_a{}^4}{12}$$
$$(2.581 \times 10^7) = \frac{w_a{}^4}{12}$$
$$w_a = 132.7\ mm$$

$$I = \frac{w_b{}^4}{12}$$
$$(1.203 \times 10^7) = \frac{w_a{}^4}{12}$$
$$w_b = 109.6\ mm$$

Before confirming these dimensions, we determine the dimension requirements to avoid yielding and select the greatest of the two. A concrete compressive strength of 50 MPa (Atkins, 2020), and the critical load from the previous step is used to determine required yielding dimensions.

$$f_c' = \frac{P_{cr}}{A}$$
$$50\ MPa = \frac{638.599\ kN}{w_a{}^2}$$
$$w_a = 113.0\ mm$$

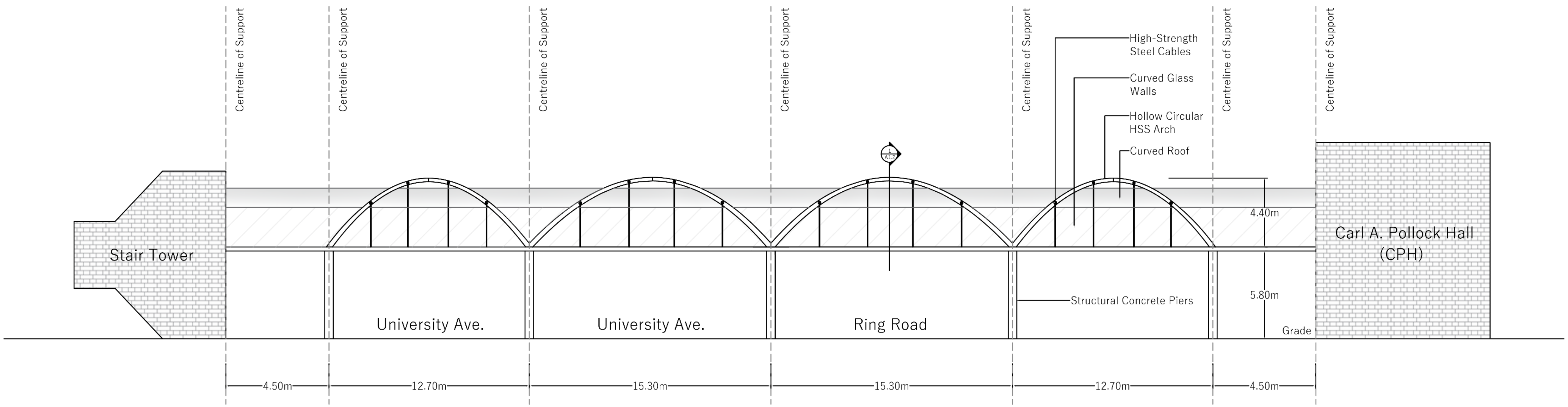
$$f_c' = \frac{P_{cr}}{A}$$
$$50\ MPa = \frac{1029.99\ kN}{w_b{}^2}$$
$$w_b = 143.5\ mm$$

The arms will buckle before they yield, thus the first calculated dimensions can be kept. However, the base will actually yield before it buckles, and thus its dimensions will be governed by yielding failure instead.

All in all, the arms must have a cross-section greater than 132.7 mm x 132.7 mm, while the piers must be greater than 143.5 mm x 143.5 mm in size.

Appendix B: Drawing Package

Drawing Set



Pierre Mendes Consultants

PROJECT
UNIVERSITY AVE. PEDESTRIAN OVERPASS

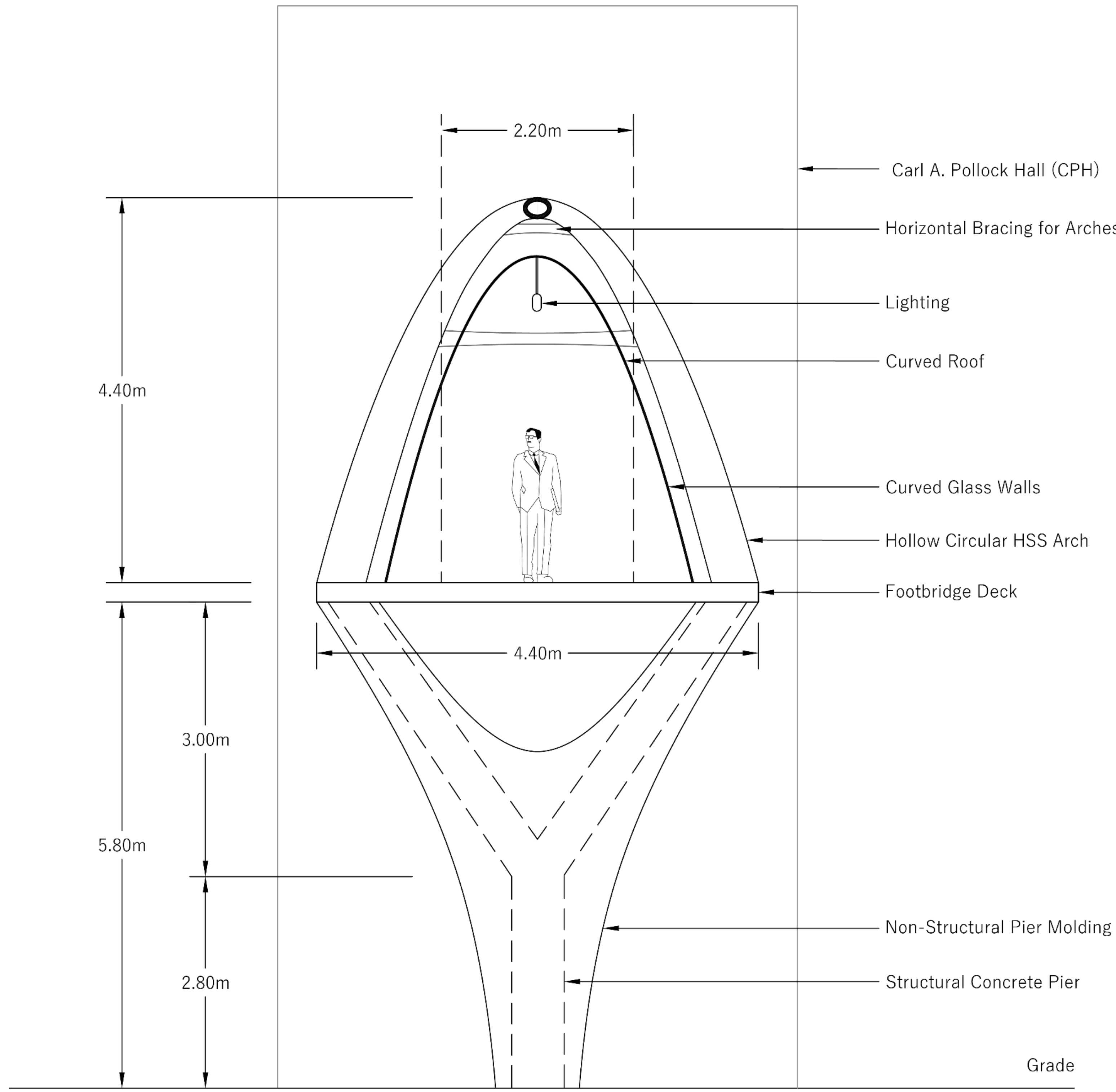
LOCATION **Waterloo ON, Canada**
DATE **18 December 2020**
ENGINEERING **Mapendo Ngilinga de Carvalho**
Nicolas Gomes

NOTES

SCALE **1:250**

DRAWING
STRUCTURAL ELEVATION NORTH-EAST SIDE

SHEET NUMBER
A1.1



Pierre Mendes Consultants

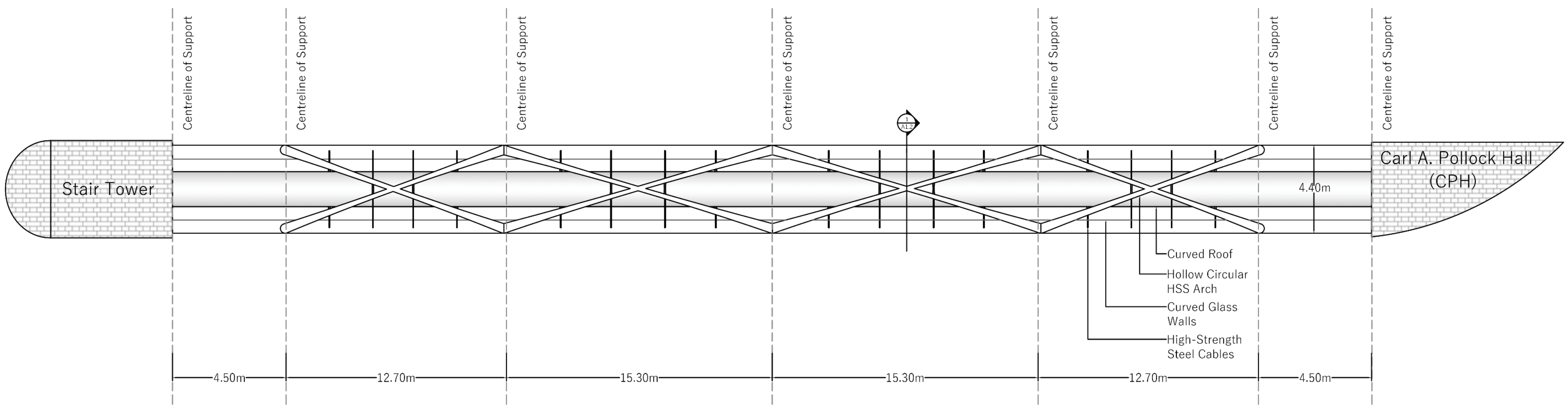
PROJECT
UNIVERSITY AVE. PEDESTRIAN OVERPASS

LOCATION **Waterloo ON, Canada**
DATE **18 December 2020**
ENGINEERING **Mapendo Ngilinga de Carvalho**
Nicolas Gomes

NOTES

SCALE **1:50**
DRAWING
SECTION ELEVATION
FULL HEIGHT

SHEET NUMBER
A1.2



Pierre
Mendes
Consultants

PROJECT
UNIVERSITY AVE. PEDESTRIAN OVERPASS

LOCATION **Waterloo ON, Canada**
DATE **18 December 2020**

ENGINEERING **Mapendo Ngilinga de Carvalho**
Nicolas Gomes

NOTES

SCALE **1:250**

DRAWING
STRUCTURAL PLAN
TOP VIEW

SHEET NUMBER
A2.1